Solution of Random Fuzzy Differential Equation

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Abstract. Random Fuzzy Differential Equation(RFDE) describes the phenomena not only with randomness but also with fuzziness. It is widely used in fuzzy control and artificial intelligence etc. In this paper, we shall discuss RFDE as follows:

$$d\tilde{F}(t) = \tilde{f}(t, \tilde{F}(t))dt + g(t, \tilde{F}(t))dB_t,$$

where $\tilde{f}(t, \tilde{F}(t))dt$ is related to fuzzy set-valued stochastic Lebesgue integral, $g(t, \tilde{F}(t))dB_t$ is related to It$\hat{o}$ integral. Firstly we shall give some basic results about set-valued and fuzzy set-valued stochastic processes. Secondly, we shall discuss the Lebesgue integral of a fuzzy set-valued stochastic process with respect to time t, especially the Lebesgue integral is a fuzzy set-valued stochastic process. Finally by martingale moment inequality, we shall prove a theorem of existence and uniqueness of solution of random fuzzy differential equation.

Keywords: Fuzzy set-valued stochastic process, Random fuzzy differential equation, Fuzzy set-valued Lebesgue integral, Level-set process.

1 Introduction

Itô type stochastic differential equations have been widely used in the stochastic control (e.g. [5]) and financial mathematics (e.g. [17]). Random fuzzy differential equations(RFDEs) deal with the real phenomena not only with randomness but also with fuzziness. Puri and Ralescu introduced fuzzy set-valued random variable in [16], and gave the concept of differentiability by

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Hukuhara difference in [15]. Li *et al* discussed Lebesgue integral of a set-valued stochastic process with repect to time t and Lebesgue integral of a fuzzy set-valued stochastic process with repect to time t in [6] [7] [9] [10].

There are some nice papers on RFDEs. Feng [3] [4] studied mean-square fuzzy stochastic differential systems by mean-square derivative introduced in [2]. In [1], Fei proved the existence and uniqueness of the solution for RFDEs with non-Lipschitz coefficients. Malinowski proved the existence and uniqueness of the solution to RFDEs with global Lipschits-type condition in [13] and discussed local solution and global solution to RFDEs in [14]. Since the existence of Hukuhara difference is a difficult problem (c.f. [9]), we shall discuss random fuzzy differential equation with level sets and selections by martingale moment inequality in this paper.

We organize our paper as follows: in section 2, we shall introduce some necessary notations, definitions and results about set-valued stochastic processes and fuzzy set-valued stochastic processes. In section 3, we shall give Lebesgue integral of a fuzzy set-valued stochastic process with respect to time t and discuss its properties, especially the integral is a fuzzy set-valued stochastic process. Finally, we prove the theorem of existence and uniqueness of solution to RFDE.

2 Fuzzy Set-Valued Stochastic Processes

Throughout this paper, assume that $(\Omega, \mathcal{A}, \mu)$ is a complete atomless probability space, $I = [0, T]$, the σ-field filtration $\{\mathcal{A}_t : t \in I\}$ satisfies the usual conditions (i.e. containing all null sets, non-decreasing and right continuous). We assume that $\mathcal{A}$ is μ-separable as for the almost everywhere problem (cf. [9]). R is the set of all real numbers, N is the set of all natural numbers, R^d is the d-dimensional Euclidean space with usual norm $\|\cdot\|$, $\mathcal{B}(E)$ is the Borel field of the metric space E. Let $f = \{f(t), \mathcal{A}_t : t \in I\}$ be a R^d-valued adapted stochastic process. It is said that f is progressively measurable if for any $t \in I$, the mapping $(s, \omega) \mapsto f(s, \omega)$ from $[0, t] \times \Omega$ to R^d is $\mathcal{B}([0, t]) \times \mathcal{A}_t$-measurable. Each right continuous (left continuous) adapted process is progressively measurable. Assume that $\mathcal{L}^p(R^d)$ denotes the set of R^d-valued stochastic processes $f = \{f(t), \mathcal{A}_t : t \in I\}$ such that f satisfying (a) f is progressively measurable; and (b)

$$|||f|||_p = \Big[E\Big(\int_0^T \|f(t, \omega)\|^p ds\Big)\Big]^{1/p} < \infty.$$

Let $f, f' \in \mathcal{L}^p(R^d)$, $f = f'$ if and only if $|||f - f'|||_p = 0$. Then $(\mathcal{L}^p(R^d), |||\cdot|||_p)$ is complete. Now we review notation and concepts of set-valued stochastic processes. Assume that $\mathbf{K}(R^d)$ is the family of all nonempty, closed subsets of R^d, and $\mathbf{K}_c(R^d)$ (*resp.* $\mathbf{K}_k(R^d)$, $\mathbf{K}_{kc}(R^d)$) is the family of all nonempty closed convex (*resp.* compact, compact convex) subsets of R^d.